

Non-anticommutative Deformation of Effective Potentials in Supersymmetric Gauge Theories ¹

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Abstract

We studied a nilpotent Non-Anti-Commutative (NAC) deformation of the effective superpotentials in supersymmetric gauge theories, caused by a constant self-dual graviphoton background. We derived the simple non-perturbative formula applicable to any NAC (star) deformed chiral superpotential. It is remarkable that the deformed superpotential is always ‘Lorentz’-invariant. As an application, we considered the NAC deformation of the pure super-Yang-Mills theory whose IR physics is known to be described by the Veneziano-Yankielowicz superpotential (in the undeformed case). The unbroken gauge invariance of the deformed effective action gives rise to severe restrictions on its form. We found a non-vanishing gluino condensate in vacuum but no further dynamical supersymmetry breaking in the deformed theory.

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1 Introduction

It is widely believed that four-dimensional $N=1$ supersymmetric gauge theories are relevant to the real world physics, *cf.* the Minimal Supersymmetric Standard Model (MSSM). Amongst the important unsolved issues are confinement, a mass gap, chiral- and super-symmetry breaking. They are all related to the well known fact that the effective description of infra-red physics in a non-abelian gauge theory is usually a strong coupling problem. In the absence of an analytic proof of confinement and chiral symmetry breaking, it is quite natural to assume them and then figure out the effective action, as is usual in the standard QCD. For a review of the nonperturbative dynamics in the supersymmetric gauge theories, see ref. [1]. Our motivation behind this paper was a search for a dynamical supersymmetry breaking by certain supergravitational corrections to the supersymmetric gauge theory, caused by non-anticommutativity (NAC) of the fermionic coordinates in superspace [2, 3, 4]. To the best of our knowledge, even an exact (non-perturbative) NAC deformation of the non-polynomial effective potentials was never calculated in the past, which is the necessary pre-requisite to their truly non-perturbative physical applications. The NAC itself is known to break supersymmetry by half [5], so our problem here is first to compute the NAC-deformation of any superpotential and then to study whether supersymmetry could be dynamically broken.

Details are dependent upon the matter content of a gauge theory. In this paper we only consider the *pure* $N=1$ Super-Yang-Mills (SYM) theory with an $U(N_c) = SU(N_c) \times U(1)$ gauge group, i.e. without matter.⁶ In the IR limit the $SU(N_c)$ confines while the $U(1)$ is weakly coupled. The field content of the simplest confining theory (called the $N=1$ supersymmetric gluodynamics) is given by gluons and their fermionic partners – gluinos, all in the adjoint representation of the gauge group. The low-energy effective action of that theory is known since 1982 [7].

Our paper is organized as follows. In sect. 2 we give our notation, as regards the quantum $N=1$ SYM theory. Sect. 3 is devoted to a brief introduction into the Non-Anti-Commutativity (NAC) along the lines of ref. [5]. It shows our setup, where we also add our comments about the NonAntiCommutativity (NAC) versus the usual NonCommutativity in field theory. The main part of our work is given by Sect. 4 where we explicitly compute the non-perturbative NAC-deformation of an arbitrary chiral superpotential, and apply our results to the standard Veneziano-Yankielowicz (VY) effective superpotential [7]. Our conclusions are given by Sect. 5. Some notation and technical details of our calculation are collected in two Appendices, A and B.

⁶We postpone adding a supersymmetric matter to another publication [6].

2 N=1 SYM in components and in superspace

The standard fundamental action of the N=1 SYM theory in components reads

$$I_{\text{SYM}} = -\frac{1}{2g^2} \int \text{Tr} [F \wedge {}^*F - 4i\bar{\lambda}\tilde{\sigma}^\mu \nabla_\mu \lambda] + \frac{\Theta}{16\pi^2} \int \text{Tr} F \wedge F, \quad (2.1)$$

where we have introduced the YM Lie algebra-valued two-form $F = F_{\mu\nu} dx^\mu \wedge dx^\nu$ and the chiral spinors, λ and $\bar{\lambda}$. The *F stands for the Poincaré-dual two-form of F . We use the standard (in supersymmetry) two-component notation for spinors [8]. It is common to unify the YM coupling constant g and the theta-parameter Θ into a single (complex) coupling constant

$$\tau = \text{Re } \tau + i \text{Im } \tau = \frac{4\pi}{g^2} - i \frac{\Theta}{2\pi}. \quad (2.2)$$

We study either *Euclidean* or *Atiyah-Ward* versions of the SYM theory in $4+0$ or $2+2$ dimensions, respectively [9]. The last term in eq. (2.1) can be identified with the quantized instanton charge (in Euclidean space)

$$Q_{\text{instanton}} = \frac{1}{16\pi^2} \int \text{Tr} F \wedge F \in \mathbf{Z}. \quad (2.3)$$

The ‘Lorentz’ group factorizes in Euclidean space, $SO(4) \cong SU(2)_L \times SU(2)_R$, as well as in Atiyah-Ward space, $SO(2,2) \cong SU(1,1)_L \times SU(1,1)_R$, so that the ‘left’ and ‘right’ chiral spinors, λ and $\bar{\lambda}$, are fully independent upon each other (being also real in $2+2$ dimensions) [9].

Since the theory is supersymmetric, it is better to make supersymmetry manifest and thus make sure that quantum theory is also supersymmetric. It can be done in superspace $(x^\mu, \theta^\alpha, \bar{\theta}^{\dot{\alpha}})$, where $\mu = 1, 2, 3, 4$ and $\alpha = 1, 2$. Here θ^α and $\bar{\theta}^{\dot{\alpha}}$ are the anticommuting (Grassmann) spinor coordinates of superspace, whereas x^μ stand for the usual bosonic (commuting) coordinates in \mathbf{R}^4 .

The superspace extension of a YM field A_μ is given by a general, Lie algebra-valued N=1 scalar superfield $V(x, \theta, \bar{\theta})$ subject to the supergauge transformations,

$$e^V \rightarrow e^{V'} = e^{-i\bar{\Lambda}} e^V e^{i\Lambda}, \quad (2.4)$$

with the gauge parameter Λ being a chiral superfield, i.e. $\bar{D}_\bullet \Lambda = 0$. The spinorial supercovariant derivatives in superspace, D_α and $\bar{D}_{\dot{\alpha}}$, are supposed to commute with spacetime translations and supersymmetry generators, by their definition. See ref. [8] and Appendix A for more details.

The superfield analogue of the YM field strength is given by the N=1 chiral gauge-covariant superfield strength (of canonical dimension 3/2),

$$W_\alpha = -\frac{1}{4}\bar{D}^2 (e^{-V} D_\alpha e^V) , \quad (2.5a)$$

and its N=1 anti-chiral cousin,

$$\bar{W}_{\dot{\alpha}} = \frac{1}{4}D^2 (e^V \bar{D}_{\dot{\alpha}} e^{-V}) . \quad (2.5b)$$

It is useful to introduce a chiral basis in superspace, where chirality is manifest, by shifting the bosonic coordinates as

$$y^\mu = x^\mu + i\theta^\alpha \sigma^\mu_{\alpha\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} . \quad (2.6)$$

Then any chiral superfield Φ is simply a function $\Phi(y, \theta)$. The spinorial covariant derivatives in the chiral basis are given by

$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} + 2i\sigma^\mu_{\alpha\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} \frac{\partial}{\partial y^\mu} , \quad \bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} . \quad (2.7)$$

They obey an algebra

$$\{D_\alpha, D_\beta\} = 0 , \quad \{\bar{D}_{\dot{\alpha}}, \bar{D}_{\dot{\beta}}\} = 0 , \quad \{D_\alpha, \bar{D}_{\dot{\alpha}}\} = -2i\sigma^\mu_{\alpha\dot{\alpha}} \partial_\mu . \quad (2.8)$$

The N=1 SYM action in superspace reads

$$\begin{aligned} I_{SYM, s} &= \frac{\tau}{16\pi} \int d^4 y d^2 \theta \text{Tr} W^\alpha W_\alpha + \frac{\bar{\tau}}{16\pi} \int d^4 \bar{y} d^2 \bar{\theta} \text{Tr} \bar{W}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}} \\ &= I_{SYM} + \frac{1}{g^2} \int d^4 x \text{Tr} D^2 , \end{aligned} \quad (2.9)$$

where D is the auxiliary field (of dimension 2) needed to close the algebra of the supersymmetry transformations on the SYM fields, while I_{SYM} is given by eq. (2.1). The auxiliary field D decouples, while its equation of motion is $D = 0$ (this feature is not so obvious when considering the highly non-linear effective actions in superspace).

The gauge freedom (2.4) can be used to get rid of some of the field components of the gauge superfield V , by putting it into the form

$$V(y, \theta, \bar{\theta}) = -(\theta^\mu \bar{\theta}) A_\mu(y) + i\theta^2 \bar{\theta}_{\dot{\alpha}} \bar{\lambda}^{\dot{\alpha}}(y) - i\bar{\theta}^2 \theta^\alpha \lambda_\alpha(y) + \frac{1}{2}\theta^2 \bar{\theta}^2 [D(y) - i\partial \cdot A(y)] , \quad (2.10)$$

without breaking supersymmetry, thus rendering V to obey the nilpotency condition $V^3 = 0$. This is known as the Wess-Zumino gauge [8]. Substituting (2.10) into (2.5a) yields

$$W_\alpha(y, \theta) = -i\lambda_\alpha + [\delta_\alpha^\beta D - i(\sigma^{\mu\nu})_\alpha{}^\beta F_{\mu\nu}] \theta_\beta + \theta^2 \sigma^\mu_{\alpha\dot{\alpha}} \nabla_\mu \bar{\lambda}^{\dot{\alpha}} . \quad (2.11)$$

The classical SYM action is not only supersymmetric, it is also scale and chirally invariant, because of the absence of dimensional parameters and the ‘left’-‘right’ symmetry. The left-right symmetry commutes with supersymmetry, while it should not be confused with the chiral R-symmetry [8] that does not commute with supersymmetry. In quantum theory, the scale invariance and the R-symmetry are broken due to anomalies. Supersymmetry is expected to be preserved, while the left-right symmetry is expected to be violated too.

The one-loop renormalization group (RG) beta-function of the theory (2.1) can be computed by the standard procedure of quantum field theory, with the well known result

$$\beta(g) = \mu \frac{dg}{d\mu} = -\frac{3N_c g^3}{16\pi^2} , \quad (2.12)$$

where the RG scale μ and the running coupling constant $g(\mu)$ have been introduced. The negative sign on the r.h.s. of eg. (2.12) implies the UV asymptotic freedom as well as the strong coupling in the IR limit. The anomalous trace of the stress-energy tensor and the anomalous divergence of the axial current are also well known (see e.g., ref. [7]),

$$T_\mu^\mu = \frac{\beta(g)}{2g} (F_{\mu\nu}^a)^2 , \quad (2.13a)$$

and

$$\partial_\mu J_5^\mu = -\frac{\beta(g)}{2g} F_{\mu\nu}^a * F_{\mu\nu}^a . \quad (2.13b)$$

The supercurrent $(S_\mu^\alpha, \bar{S}_\mu^{\dot{\alpha}})$ is conserved, but it is subject to the superconformal anomaly,

$$\tilde{\sigma}^{\mu\dot{\alpha}\beta} S_{\mu\beta} = \frac{\beta(g)}{g} F_{\mu\nu}^a (\tilde{\sigma}_{\mu\nu})_{\dot{\alpha}}^{\dot{\gamma}} \bar{\lambda}^{a\dot{\gamma}} , \quad (2.13c)$$

and similarly for $\bar{S}_\mu^{\dot{\alpha}}$. Both the currents and their anomalies are known to form supermultiplets [10, 11]. In particular, the stress-tensor T_μ^ν , the axial current J_5^μ and the supercurrent S_μ^α belong to the field components of a constrained vector superfield $T_{\alpha\dot{\alpha}}^{\dot{\bullet}}$ subject to the classical relations

$$D^\alpha T_{\alpha\dot{\alpha}}^{\dot{\bullet}} = \bar{D}^{\dot{\alpha}} T_{\alpha\dot{\alpha}}^{\dot{\bullet}} = 0 . \quad (2.14)$$

The anomalies form a chiral superfield \mathcal{T} with $\bar{D}_{\dot{\alpha}} \mathcal{T} = 0$ [11]. The classical relations (2.14) get modified in the quantum SYM theory as

$$\bar{D}^{\dot{\alpha}} T_{\alpha\dot{\alpha}}^{\dot{\bullet}} \propto D_\alpha \mathcal{T} , \quad \text{and} \quad D^\alpha T_{\alpha\dot{\alpha}}^{\dot{\bullet}} \propto \bar{D}_{\dot{\alpha}} \bar{\mathcal{T}} . \quad (2.15)$$

In part, the anticommutator of a supersymmetry charge Q with the supercurrent results in the conformal anomaly proportional to F^2 . Taking the vacuum expectation

value of that relation implies that a non-vanishing value of $\langle F^2 \rangle$ gives rise to $\langle T_\mu^\mu \rangle \neq 0$ and $Q|0\rangle \neq 0$, i.e. it results in a *Dynamical Supersymmetry Breaking* (DSB). See a review [12] for more details about the DSB.

In supersymmetry $\text{Tr } F^2$ and $\text{Tr }^* FF$ are united into a complex field that belongs to a chiral supermultiplet, together with the gaugino composite field $\text{Tr } (\lambda^\alpha \lambda_\alpha)$. It is known as the N=1 chiral *glueball* superfield (all traces here are taken in the $SU(N_c)$)

$$S \propto \text{Tr } (W^\alpha W_\alpha) . \quad (2.16)$$

The r.h.s. of eq. (2.16) is of mass dimension 3, while eq. (2.13) gives the natural normalization factor $\beta(g)/2g$, as in ref. [7]. We are going to use a *dimensionless* glueball superfield S by further rescaling $S \rightarrow S/\mu^3$, where μ is the RG scale.

Next, though the classical glueball superfield is nilpotent due to the fermionic statistics of gluons, $S^{N_c^2} = 0$, it is not necessarily true in quantum theory since the nilpotency condition is subject to quantum corrections. We ignore this subtlety in what follows.

The field components of the glueball superfield S are given by (up to a constant)

$$\text{Tr}(\lambda^2) \equiv \phi , \quad \text{Tr} \left[\frac{i}{2} (\sigma^{\mu\nu})_\alpha{}^\beta F_{\mu\nu} \lambda_\beta \right] \equiv \chi_\alpha , \quad \text{Tr}(F_{\mu\nu} F^{\mu\nu} + i F_{\mu\nu}^* F^{\mu\nu}) \equiv M. \quad (2.17)$$

The glueball superfield S is a singlet (colorless) with respect to the gauge group, so that it appears to the natural *order* parameter in any description of the IR physics of the N=1 supersymmetric gluodynamics. It is usually assumed that the IR physics can be described by some effective action depending upon the chiral glueball superfield S and its anti-chiral cousin \bar{S} , while both are to be considered as the independent superfields in the IR limit [7, 13]. In particular, M is going to play the role of the auxiliary field in what follows, while the DSB occurs whenever $\langle M \rangle \neq 0$.

The most general, manifestly supersymmetric low-energy effective action (without higher derivatives of the field components in eq. (2.17)) is given by

$$I[S, \bar{S}] = \mu^2 \int d^4x d^2\theta d^2\bar{\theta} K(S, \bar{S}) + \mu^3 \int d^4y d^2\theta V(S) + \mu^3 \int d^4\bar{y} d^2\bar{\theta} \bar{V}(\bar{S}) , \quad (2.18)$$

where the dimensionless kinetic function $K(S, \bar{S})$ is called a *Kähler* potential, and another dimensionless function $V(S)$ is called a *superpotential*. In Minkowski spacetime the function $\bar{V}(\bar{S})$ is simply the Hermitean conjugate of $V(S)$, though it is not going to be the case in sects. 3 and 4, where either Euclidean or Atiyah-Ward spacetime signatures are assumed.

As was first discovered in ref. [7], there exists a unique non-perturbative scalar superpotential (nowadays famously known as the VY superpotential) that reproduces the anomaly structure of the N=1 SYM and that of the N=1 gluodynamics, namely,

$$V_{\text{VY}}(S) = N_c S \ln S + \tau_{\text{ren}} S , \quad (2.19)$$

where we have introduced the renormalized value τ_{ren} of the SYM coupling constant at the scale μ . For instance, in one-loop we have

$$\tau_{\text{ren}} = \tau_0 + 3N_c \ln \frac{\mu}{\mu_0} , \quad (2.20)$$

with μ_0 being the scale where the bare coupling τ_0 is defined. It is worth mentioning that no dimensional quantities appear in eq. (2.19). There are other ways of ‘deriving’ the VY superpotential, either from the field theory [13, 14] or from the matrix models [15] by using the Dijkgraaf-Vafa correspondence [16].

Minimizing the VY superpotential, $V'(S) = 0$, one finds a non-vanishing gluino condensate,

$$\langle \text{Tr} \lambda^2 \rangle \propto \langle S \rangle \propto e^{-\tau_{\text{ren}}/N_c} \propto e^{-4\pi^2/g^2 N_c} , \quad (2.21)$$

but no dynamical susy breaking (DSB) because of $\langle M \rangle = \langle \text{Tr} F^2 \rangle = \langle \text{Tr} F^* F \rangle = 0$.

3 Nilpotent NAC deformation

The Non-Anti-Commutative (NAC) deformation of the N=1 superspace is given by [2, 3]

$$\{\theta^\alpha, \theta^\beta\} = C^{\alpha\beta} , \quad (3.1)$$

where $C^{\alpha\beta}$ are some constants, i.e. the chiral spinorial superspace coordinates are no longer Grassmann but satisfy a Clifford algebra (3.1). It is consistent to keep unchanged the rest of the (anti)commutation relations between the N=1 superspace coordinates (in the chiral basis),

$$[y^\mu, y^\nu] = [y^\mu, \theta^\alpha] = [y^\mu, \bar{\theta}^{\dot{\alpha}}] = 0 , \quad (3.2)$$

as well as

$$\{\theta^\alpha, \bar{\theta}^{\dot{\beta}}\} = \{\bar{\theta}^{\dot{\alpha}}, \bar{\theta}^{\dot{\beta}}\} = 0 , \quad (3.3)$$

either in Euclidean or Atiyah-Ward spacetime where θ^α and $\bar{\theta}^{\dot{\alpha}}$ are truly independent. This choice of a NAC deformation is sometimes called *nilpotent* [17, 18] since it gives rise to a *local* deformed field theory. The physical significance of the NAC deformation

(3.1) in string theory was uncovered by Ooguri and Vafa [4]. They argued that the $C^{\alpha\beta}$ can be thought of as the vacuum expectation values of the self-dual graviphoton field strength $F_{\text{graviphoton}}^{\mu\nu}$ (see also refs. [19, 5]) with

$$(\alpha')^2 F_{\text{graviphoton}}^{\alpha\beta} = C^{\alpha\beta} \quad . \quad (3.4)$$

In the Calabi-Yau (CY) compactified type-IIB superstrings a self-dual RR-type 5-form can have a non-vanishing flux over certain CY cycles, which gives rise to a non-vanishing self-dual graviphoton flux in four dimensions. From the viewpoint of the N=1 SYM theory in four dimensions, the deformation (3.1) can be thought of as the result of some gravitational corrections coming after embedding the gauge theory into extended supergravity or superstrings.

The $C^{\alpha\beta} \neq 0$ in eq. (3.1) explicitly breaks the four-dimensional ‘Lorentz’ invariance at the fundamental level. The NAC nature of θ ’s can be fully taken into account in field theory by using the Moyal-Weyl-type star product (functions of θ ’s are to be ordered) [5]

$$f(\theta) \star g(\theta) = f(\theta) \exp \left(-\frac{C^{\alpha\beta}}{2} \frac{\overleftarrow{\partial}}{\partial\theta^\alpha} \frac{\vec{\partial}}{\partial\theta^\beta} \right) g(\theta) \quad (3.5)$$

that clearly respects the N=1 superspace chirality, so that the star product of any two chiral or anti-chiral superfields is again a chiral or anti-chiral superfield, respectively.

The star product (3.5) is polynomial in the deformation parameter,⁷

$$f(\theta) \star g(\theta) = fg + (-1)^{\deg f} \frac{C^{\alpha\beta}}{2} \frac{\partial f}{\partial\theta^\alpha} \frac{\partial g}{\partial\theta^\beta} - \det C \frac{\partial^2 f}{\partial\theta^2} \frac{\partial^2 g}{\partial\theta^2} \quad , \quad (3.6)$$

where we have used the identities

$$\det C = \frac{1}{2} \varepsilon_{\alpha\gamma} \varepsilon_{\beta\delta} C^{\alpha\beta} C^{\gamma\delta} = \frac{1}{4} (C_{\mu\nu})^2 \quad , \quad (3.7)$$

and the standard notation between the vector and spinor indices,

$$C^{\mu\nu} = C^{\alpha\beta} \varepsilon_{\beta\gamma} (\sigma^{\mu\nu})_\alpha{}^\gamma \quad , \quad (3.8)$$

with $\mu, \nu = 1, 2, 3, 4$ and $\alpha, \beta, \dots = 1, 2$. Any value of the scalar $\det C$ is obviously ‘Lorentz’-invariant, while it can be real in $2 + 2$ dimensions.

The nilpotent NAC deformation versus the spacetime Non-Commutativity (NC) [20, 21], described by the relations $[y^\mu, y^\nu] = iB^{\mu\nu} \neq 0$, has several advantages. In particular, the nilpotent NAC does not lead to a non-local field theory but to a very

⁷See Appendix A for more about our notation.

limited (finite) number of the new vertices. This also implies the absence of the UV/IR mixing problem common to all NC theories. The nilpotent NAC deformation of an abelian supersymmetric gauge theory is almost trivial [18], e.g., there are no $U(1)$ monopoles and instantons there. Of course, there are also serious problems related to NAC. For instance, the nilpotent NAC deformation is only possible in Euclidean or Atiyah-ward spacetimes, it leads to non-Hermitean actions (see below), which may cause problems with unitarity. However, a discussion of unitarity in the NAC deformed field theories is beyond the scope of this paper.

The NAC deformation (3.1) of the $N=1$ SYM theory (2.1) with the $U(N_c)$ gauge group in Euclidean space was considered by Seiberg [5]. We refer to his paper [5] for details, but we would like to emphasize here some aspects of ref. [5] that are going to be relevant for our next sect. 4. In particular, the deformation (3.1) breaks just half of $N=1$ or $(\frac{1}{2}, \frac{1}{2})$ supersymmetry, while another half of supersymmetry remains unbroken [5].

The fate of the gauge invariance in a quantized NAC-deformed SYM theory did not receive enough attention in ref. [5], and it remains to be a highly non-trivial issue. When requiring the component fields to transform in the standard (undeformed) way under the gauge transformations, the NAC gauge superfield in the WZ gauge has to be modified [5],⁸

$$V_C(y, \theta, \bar{\theta}) = V(y, \theta, \bar{\theta}) - \frac{i}{4} \bar{\theta}^2 \theta^\alpha \varepsilon_{\alpha\beta} C^{\beta\gamma} \sigma_{\gamma\dot{\gamma}}^\mu \{ \bar{\lambda}^{\dot{\gamma}}, A_\mu \}, \quad (3.9)$$

where $V(y, \theta, \bar{\theta})$ is given by eq. (2.10). By construction [5], the *residual* gauge transformations (keeping the form of eq. (3.9) intact) of the field components $(A_\mu, \lambda^\alpha, \bar{\lambda}^{\dot{\alpha}}, D)$ are C -independent, i.e. of the standard form. It is straightforward to calculate the deformed gauge superfield strengths (in the WZ gauge). One finds [5]

$$(W_C)_\alpha(y, \theta) = W_\alpha(y, \theta) + \varepsilon_{\alpha\gamma} C^{\gamma\beta} \theta_\beta \bar{\lambda}^2 \quad (3.10)$$

and

$$\begin{aligned} (\bar{W}_C)_{\dot{\alpha}}(\bar{y}, \bar{\theta}) = & \bar{W}_{\dot{\alpha}}(\bar{y}, \bar{\theta}) - \bar{\theta}^2 \left(\frac{1}{2} C^{\mu\nu} \{ F_{\mu\nu}, \bar{\lambda}_{\dot{\alpha}} \} + C^{\mu\nu} \left\{ A_\nu, \nabla_\mu \bar{\lambda}_{\dot{\alpha}} - \frac{i}{4} [A_\mu, \bar{\lambda}_{\dot{\alpha}}] \right\} \right. \\ & \left. + \frac{1}{4} (\det C) \left\{ \bar{\lambda}^2, \bar{\lambda}_{\dot{\alpha}} \right\} \right). \end{aligned} \quad (3.11)$$

Hence, the NAC chiral superfield strength (3.10) is gauge-covariant, whereas the NAC anti-chiral superfield strength (3.11) is not as long as $C \neq 0$. Moreover, both eqs. (3.9) and (3.11) contain anti-commutators of the Lie algebra-valued gauge fields, whose closure restricts the choice of a gauge group (e.g., the $U(N_c)$ is ok).

⁸All the deformed quantities vs. the undeformed ones are marked by the subscript C .

As regards the fundamental SYM Lagrangian in eq. (2.9), it is given by a linear combination of two terms,

$$\int d^2\theta \operatorname{Tr} W^2 \quad \text{and} \quad \int d^2\bar{\theta} \operatorname{Tr} \bar{W}^2 \quad . \quad (3.12)$$

Their NAC deformations are almost the same, up to a total derivative in \mathbf{R}^4 [5],

$$\int d^2\theta \operatorname{Tr}(W^2)_C = \int d^2\theta \operatorname{Tr} W^2 - iC^{\mu\nu} \operatorname{Tr}(F_{\mu\nu} \bar{\lambda}^2) + (\det C) \operatorname{Tr}(\bar{\lambda}^2)^2 \quad (3.13)$$

and

$$\int d^2\bar{\theta} (\operatorname{Tr} \bar{W}^2)_C = \int d^2\bar{\theta} \operatorname{Tr} \bar{W}^2 - iC^{\mu\nu} \operatorname{Tr}(F_{\mu\nu} \bar{\lambda}^2) + (\det C) \operatorname{Tr}(\bar{\lambda}^2)^2 + \text{total derivative} \quad . \quad (3.14)$$

Our calculation of the total derivative in eq. (3.14) reveals that it is not gauge-invariant, *viz.*

$$\text{total derivative} = \partial_\mu (C^{\mu\nu} \operatorname{Tr} A_\nu \bar{\lambda}^2) \quad . \quad (3.15)$$

When considering the effective field theory (2.18) originating from the fundamental supersymmetric gauge theory, eqs. (3.13), (3.14) and (3.15) imply that only the *holomorphic* superpotential can be affected by the NAC-deformation, whereas the anti-holomorphic superpotential cannot (up to a linear contribution), as long as the gauge invariance is preserved in quantum theory (see also refs. [22, 23]).

As regards quantum properties of the NAC-deformed SYM theory, it was argued [24] that it is still renormalizable to all orders of perturbation theory despite the presence of apparently non-renormalizable (by power counting) C -dependent interactions. The gauge invariance of a quantized $N = \frac{1}{2}$ SYM theory at one-loop was proved in ref. [25]. The gauge-invariant RG beta-function and the anomalies of the NAC deformed SYM theory are apparently *the same* as that of the undeformed theory (see also ref. [26] for explicit one-loop calculations).

It is also interesting to see what happens to the $U(1)$ factor of the gauge group $U(N_c) = SU(N_c) \times U(1)$.⁹ In the superspace approach of ref. [5] the $U(N_c)$ gauge transformations are NAC-deformed, while the $U(1)$ factor is necessary for a closure of the gauge algebra. In the WZ gauge [5] the $U(N_c)$ gauge transformations are undeformed (i.e. C -independent), the $U(1)$ and $SU(N_c)$ gauge transformations can be separated, but there is a coupling between photinos and gluinos due to the last term in eq. (3.13). We assume, however, that the NAC deformed SYM is still well-defined in the IR, where the $SU(N_c)$ confines and the $U(1)$ is weakly coupled like that in the undeformed theory.

⁹We are grateful to G. Silva and S. Terashima for discussions about this point.

4 NAC-deformed effective potentials

We begin with our main result of this paper, which is the beautiful formula describing the non-perturbative NAC (star) deformation of an *arbitrary* superpotential $V(f)$,

$$\begin{aligned}
\int d^2\theta \, V_\star(f) &= V'(\phi)M - \frac{1}{2}V''(\phi)\chi^2 \\
&+ \sum_{k=1}^{\infty} \frac{1}{(2k+1)!} (-\det C)^k M^{2k} (V^{(2k+1)}(\phi)M - \frac{1}{2}V^{(2k+2)}(\phi)\chi^2) \\
&= \frac{1}{2c} \{V(\phi + cM) - V(\phi - cM)\} \\
&- \frac{\chi^2}{4cM} \{V'(\phi + cM) - V'(\phi - cM)\} \, ,
\end{aligned} \tag{4.1}$$

where we have used the notation (A.6) for the field components of a chiral superfield $f(y, \theta)$, and have introduced the effective (‘Lorentz’-invariant) deformation parameter

$$\sqrt{-\det C} \equiv c \, . \tag{4.2}$$

The primes denote the derivatives of the function V with respect to its argument. The star subscript means that all the products of f ’s (in Taylor expansion of $V(f)$) are to be taken by using the star product (3.5).

It is worth mentioning that eq. (4.1) is ‘Lorentz’-invariant. In particular, as regards the purely bosonic terms, eq. (4.1) yields the remarkably simple non-perturbative equation,

$$\left. \int d^2\theta \, V_\star(f) \right|_{\text{bosonic}} = \frac{1}{2c} \{V(\phi + cM) - V(\phi - cM)\} \tag{4.3}$$

which clearly shows that the NAC deformation of any potential V just amounts to the M -dependent splitting of the leading argument of the potential (after integrating over the fermionic coordinates).

Our equation (4.1) agrees with the earlier calculations [22] in the case of a cubic superpotential (i.e. the NAC deformed supersymmetric Wess-Zumino model).¹⁰

We found eq. (4.1) as a result of our complicated calculations that we now briefly describe. It is clearly enough to prove eq. (4.1) in the case of a power -like superpotential, $V(f) = f^p$, with some positive integer p . A straightforward but tedious application of the rule (3.5) by induction yields

$$f_\star^p = f^p + \sum_{j=0}^{[\frac{p-2}{2}]} A_{j+1}^{(p)} \, , \tag{4.4}$$

¹⁰When preparing our paper for publication we learned that some perturbative calculations of the NAC star-deformed action (2.18) also appeared in ref. [27].

where we have introduced the notation

$$A_{j+1}^{(p)} = (-\det C)^{j+1} \sum_{k_1=1}^{p-j} \sum_{k_2=1}^{p-j-k_1} \cdots \sum_{k_{j+1}=1}^{p-j-\sum_{r=1}^j k_r} \frac{\partial^2}{\partial \theta^2} \left(f^{p-j-\sum_{s=1}^{j+1} k_s} \right) \times \\ \times \left(\frac{\partial^2 f}{\partial \theta^2} \right)^{j+1} \frac{\partial^2 f^{k_{j+1}}}{\partial \theta^2} \frac{\partial^2 f^{k_j}}{\partial \theta^2} \cdots \frac{\partial^2 f^{k_2}}{\partial \theta^2} f^{k_1-1} . \quad (4.5)$$

Integrating over θ 's or, equivalently, taking the last field component of the chiral superfield (4.5) gives rise to many cancellations, with the result (e.g., when $p \neq 2j+2$)

$$\int d^2 \theta A_{j+1}^{(p)} = (-\det C)^{j+1} \sum_{k_1=1}^{p-j} \sum_{k_2=1}^{p-j-k_1} \cdots \sum_{k_{j+1}=1}^{p-j-\sum_{r=1}^j k_r} \\ \int d^2 \theta \frac{\left(p-j-\sum_{s=1}^{j+1} k_s \right) (k_1-1) k_2 \cdots k_{j+1}}{p-2(j+1)} f^{p-2(j+1)} \left(\frac{\partial^2 f}{\partial \theta^2} \right)^{2(j+1)} . \quad (4.6a)$$

When $p = 2j+2$ and thus even, we found the very simple formula,

$$A_{p/2}^{(p)} = (-\det C)^{p/2} \left(\frac{\partial^2 f}{\partial \theta^2} \right)^p , \quad \text{so that} \quad \int d^2 \theta A_{p/2}^{(p)} = 0 . \quad (4.6b)$$

After combining eqs. (4.4), (4.5) and (4.6) we find

$$\int d^2 \theta f_\star^p = \int d^2 \theta f^p \\ + \sum_{j=0}^{\lfloor \frac{p-2}{2} \rfloor} (-\det C)^{j+1} \frac{p(p-1) \cdots (p-2j-1)}{(2j+3)!} \int d^2 \theta f^{p-2(j+1)} \left(\frac{\partial^2 f}{\partial \theta^2} \right)^{2(j+1)} , \quad (4.7)$$

where we have also used the crucial combinatorial identity

$$\sum_{k_1=1}^{p-j} \sum_{k_2=1}^{p-j-k_1} \cdots \sum_{k_{j+1}=1}^{p-j-\sum_{r=1}^j k_r} (p-j-\sum_{s=1}^{j+1} k_s) (k_1-1) k_2 \cdots k_{j+1} = \binom{p}{2j+3} . \quad (4.8)$$

We refer to Appendix B for further details, as regards eq. (4.8).

We are now prepared to discuss the component structure of the NAC deformed effective action (2.18) in supersymmetric gauge theories. As regards the Kähler term in eq. (2.8), we do not have any control of it, so we are going to proceed with a generic (undeformed) effective Kähler metric

$$G(\phi, \bar{\phi}) = \partial_\phi \bar{\partial}_{\bar{\phi}} K(\phi, \bar{\phi}) . \quad (4.9)$$

For example, as regards the VY effective action of the N=1 SYM, one has [7]

$$K(\phi, \bar{\phi}) = (S\bar{S})^{1/3} . \quad (4.10)$$

Only the chiral superpotential $V(f)$ is NAC-deformed according to eq. (4.1), whereas the anti-chiral effective superpotential $\bar{V}(\bar{f})$ is not deformed at all. In the VY case, the latter takes the same form (2.19).

The bosonic terms contributing to a generic NAC-deformed scalar potential W (in components) are thus given by (we assume that $\langle \chi^2 \rangle = 0$)

$$-W = G(\phi, \bar{\phi}) M \bar{M} + \bar{V}'(\bar{\phi}) \bar{M} + \frac{1}{2c} \{V(\phi + cM) - V(\phi - cM)\} \quad . \quad (4.11)$$

The non-perturbative (algebraic) equations of motion for the auxiliary fields M and \bar{M} are easily solved as

$$\begin{aligned} M &= -\frac{1}{G} \bar{V}'(\bar{\phi}), \\ \bar{M} &= -\frac{1}{2G} \{V'(\phi + cM) + V'(\phi - cM)\} \\ &= -\frac{1}{2G} \left\{ V' \left(\phi - \frac{c}{G} \bar{V}'(\bar{\phi}) \right) + V' \left(\phi + \frac{c}{G} \bar{V}'(\bar{\phi}) \right) \right\} . \end{aligned} \quad (4.12)$$

Substituting the solution (4.12) back into eq. (4.11) gives us the scalar potential

$$W(\phi, \bar{\phi}) = \frac{1}{2c} \left\{ V \left(\phi + \frac{c}{G} \bar{V}'(\bar{\phi}) \right) - V \left(\phi - \frac{c}{G} \bar{V}'(\bar{\phi}) \right) \right\} \quad . \quad (4.13)$$

For instance, taking the limit $c \rightarrow 0$ in eq. (4.13) yields the standard equation in undeformed supersymmetry,

$$W_0 = \frac{1}{G} V'(\phi) \bar{V}'(\bar{\phi}) \quad . \quad (4.14)$$

The vacuum conditions

$$\frac{\partial W}{\partial \phi} = \frac{\partial W}{\partial \bar{\phi}} = 0 \quad (4.15)$$

in the deformed case (4.13) are given by

$$\begin{aligned} \frac{1}{G^2} \frac{\partial G}{\partial \phi} \bar{V}'(\bar{\phi}) \left\{ V' \left(\phi - \frac{c}{G} \bar{V}'(\bar{\phi}) \right) + V' \left(\phi + \frac{c}{G} \bar{V}'(\bar{\phi}) \right) \right\} \\ + \frac{1}{c} \left\{ V' \left(\phi - \frac{c}{G} \bar{V}'(\bar{\phi}) \right) - V' \left(\phi + \frac{c}{G} \bar{V}'(\bar{\phi}) \right) \right\} = 0 \end{aligned} \quad (4.16a)$$

and

$$\left[\frac{1}{G} \frac{\partial G}{\partial \bar{\phi}} \bar{V}'(\bar{\phi}) - \bar{V}''(\bar{\phi}) \right] \cdot \left[V' \left(\phi - \frac{c}{G} \bar{V}'(\bar{\phi}) \right) + V' \left(\phi + \frac{c}{G} \bar{V}'(\bar{\phi}) \right) \right] = 0 \quad , \quad (4.16b)$$

respectively. According to eq. (4.16b), there are the two possibilities:

$$\text{case A: } V' \left(\phi - \frac{c}{G} \bar{V}'(\bar{\phi}) \right) + V' \left(\phi + \frac{c}{G} \bar{V}'(\bar{\phi}) \right) = 0 \quad , \quad \text{case B: } \frac{1}{G} \frac{\partial G}{\partial \bar{\phi}} \bar{V}'(\bar{\phi}) = \bar{V}''(\bar{\phi}) \quad .$$

We now consider those two cases separately.

The case A

Taking into account the remaining equation (4.16a) immediately implies

$$V'(\phi - \frac{c}{G}\bar{V}'(\bar{\phi})) = V'(\phi + \frac{c}{G}\bar{V}'(\bar{\phi})) = 0 \quad . \quad (4.17)$$

For instance, in the case of the VY superpotential (2.19) with $G = 1$ for simplicity, one always gets a non-vanishing gluino condensate (2.21) in vacuum. As regards the expectation values of the auxiliary fields, we find on-shell

$$M = -\frac{1}{G}\bar{V}'(\bar{\phi}) = \overline{M} = 0 \quad , \quad (4.18)$$

so that dynamical supersymmetry breaking does not occur. The vacuum expectation value of the scalar potential W also vanishes.

The case B

Integration of the differential equation (case B) with respect to the unknown function G has a general solution

$$G(\phi, \bar{\phi}) = \bar{V}'(\bar{\phi})g(\phi) \quad , \quad (4.18)$$

where $g(\phi)$ is an arbitrary function. This implies that the on-shell Kähler metric is a factorizable function of ϕ and $\bar{\phi}$. By the way, it is the case for the VY Kähler potential (4.10). Equations (4.12) and (4.18) also imply on-shell

$$M = -\frac{\bar{V}'(\bar{\phi})}{G} = -\frac{1}{g(\phi)} \quad . \quad (4.19)$$

This means that Dynamical Supersymmetry Breaking (DSB) may be only possible when the equation $\bar{V}'(\bar{\phi}) = 0$ has no solutions, just like that in the undeformed supersymmetry. Unfortunately, as regards the VY superpotential, it is not the case.

5 Conclusion

We found that the nilpotent NAC deformation (3.1) gives rise to the simple non-perturbative formula (4.1), valid for any chiral superpotential. It is worth mentioning that the NAC deformed superpotential (4.1) does not break the ‘Lorentz’ invariance since the effective deformation parameter is given by the scalar (4.2), unlike that at the fundamental level where the ‘Lorentz’ invariance is manifestly broken by $C^{a\beta} \neq 0$.

As regards the supersymmetric gauge theories whose NAC deformation is known to describe some supergravitational contributions, our equation (4.1) is quite useful for an explicit computation of the non-perturbative NAC-deformed effective actions, though it cannot be used as a tool for further dynamical supersymmetry breaking. We provided some physical applications of our equation (4.1) to the NAC-deformed supersymmetric Yang-Mills theory described in the IR limit by the standard VY superpotential. We found the existence of a non-vanishing gluino condensate even after the NAC deformation. Unfortunately, we also found that no dynamical supersymmetry breaking of the remaining $N = 1/2$ supersymmetry occurs in the deformed theory.

Possible physical applications of our results are, of course, not limited to the pure SYM theory. As the most obvious extension, some number of flavours can be included [6]. Since the chiral and anti-chiral superpotentials change very differently under the NAC deformation, the apparent violation of Hermiticity of the effective action (and, perhaps, of its unitarity as well) deserves further study.

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Appendix A: About our notation

In this paper we use either Euclidean space with the signature $(+, +, +, +)$, or Atiyah-Ward space with the signature $(+, +, -, -)$, though we follow the standard notation of Wess and Bagger [8], invented for supersymmetry in Minkowski spacetime. The important differences are emphasized in the main text. Here we merely describe our book-keeping notation and the normalization conventions.

The spinorial indices are raised and lowered with the charge conjugation matrix that is diagonal in the two-component notation [8]. Our conventions for the two-dimensional Levi-Civita symbol are

$$\varepsilon_{21} = \varepsilon^{12} = 1 \quad \text{so that} \quad \varepsilon^{\alpha\beta} \varepsilon_{\beta\alpha} = 2 . \quad (A.1)$$

We use the notation

$$\theta\chi = \theta^\alpha \chi_\alpha , \quad \theta^2 = \theta^\alpha \theta_\alpha = \varepsilon_{\alpha\beta} \theta^\alpha \theta^\beta \quad (A.2)$$

so that for any two chiral spinors θ^α and χ^α we have

$$\theta^\alpha \theta^\beta = -\frac{1}{2} \varepsilon^{\alpha\beta} \theta^2 , \quad (\theta\chi)^2 = -\frac{1}{2} \chi^2 \theta^2 . \quad (A.3)$$

Our normalization of the Berezin integral over Grassmann coordinates is given by

$$\int d^2\theta \theta^2 = 1 . \quad (A.4)$$

As is well known, Grassmann integration amounts to Grassmann differentiation. We use the notation

$$\frac{\partial^2}{\partial\theta^2} = \frac{1}{4} \varepsilon^{\alpha\beta} \frac{\partial}{\partial\theta^\alpha} \frac{\partial}{\partial\theta^\beta} . \quad (A.5)$$

The field components $(\phi(y), \chi(y), M(y))$ of a chiral superfield $f(y, \theta)$ are defined by

$$f = \phi + \sqrt{2}\theta\chi + \theta^2 M , \quad \text{so that} \quad \frac{\partial^2 f}{\partial\theta^2} = M . \quad (A.6)$$

The NAC deformation (3.1) is equivalent to the star product (3.5). For instance, it is not difficult to check that eq. (3.5) implies $\theta^\alpha \star \theta^\beta + \theta^\beta \star \theta^\alpha = C^{\alpha\beta}$ indeed. The standard Grassmann rules for the fermionic coordinates of superspace also get modified [5],

$$\begin{aligned} \theta^\alpha \star \theta^\beta &= -\frac{1}{2} \varepsilon^{\alpha\beta} \theta^2 + \frac{1}{2} C^{\alpha\beta} , \\ \theta^\alpha \star \theta^2 &= C^{\alpha\beta} \theta_\beta , \\ \theta^2 \star \theta^\alpha &= -C^{\alpha\beta} \theta_\beta , \\ \theta^2 \star \theta^2 &= -\det C . \end{aligned} \quad (A.7)$$

Appendix B: more about equation (4.8)

A formal algebraic proof of the identity (4.8) is not very illuminating. So we offer here another, rather intuitive and easy proof.

First, it is not difficult to check that the left-hand-side of eq. (4.8) can be rewritten as follows:

$$\sum_{\{\sum k_i = p-j-1\}} k_1 k_2 \cdots k_{j+2} \quad \{k_i \in \mathbf{N}\}. \quad (B.1)$$

The summation is performed over all possible k_i 's such that $\sum_{i=1}^{j+2} k_i = p - j - 1$. For example, in the case of $(j, p) = (1, 8)$, we have

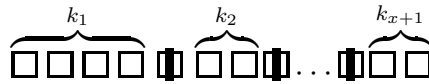
$$\begin{aligned} \sum_{\{\sum k_i = 6\}} k_1 k_2 k_3 &= 1 \cdot 1 \cdot 4 + 1 \cdot 2 \cdot 3 + 1 \cdot 3 \cdot 2 + 1 \cdot 4 \cdot 1 + 2 \cdot 1 \cdot 3 \\ &+ 2 \cdot 2 \cdot 2 + 2 \cdot 3 \cdot 1 + 3 \cdot 1 \cdot 2 + 3 \cdot 2 \cdot 1 + 4 \cdot 1 \cdot 1. \end{aligned} \quad (B.2)$$

Let's now consider an apparently unrelated problem:

let $x+1$ dots and x sticks line on $n+x$ small squares drawn in a row, where $n \geq x+1$, one item for one square, provided that each stick is between the dots. In other words, the sticks and dots have to appear alternatively on a line. The question is: how many ways of their distribution exist?

We can divide this problem into two steps. Firstly, let's put sticks in such a way that both neighboring squares of every stick are empty. Secondly, let's put dots on each breach among sticks one by one.

After putting the sticks that way, there are n empty squares remaining. Let k_i be the numbers of squares in i -th breach,



Due to the condition $k_i \geq 1$ and the fact that the sum of k_i equals to the number of all empty squares, we have $\sum_{i=1}^{x+1} k_i = n$. Once the pattern of sticks is fixed, the number of ways of putting the dots is clearly given by $\prod_{i=1}^{x+1} k_i$. Here is an example in the case of $(x, n) = (2, 6)$:

$$\begin{array}{ccccccc} \bullet & \square & | & \bullet & \square & \square & | & \bullet & & \bullet & \square & | & \bullet & \square & \square & | & \bullet & & \bullet & \square & | & \bullet & \square & \square & | & \bullet \\ \bullet & \square & | & \bullet & \square & \square & | & \bullet & & \bullet & \square & | & \bullet & \square & \square & | & \bullet & & \bullet & \square & | & \bullet & \square & \square & | & \bullet \\ \bullet & \square & | & \bullet & \square & \square & | & \bullet & & \bullet & \square & | & \bullet & \square & \square & | & \bullet & & \bullet & \square & | & \bullet & \square & \square & | & \bullet \end{array} = 2 \cdot 3 \cdot 1$$

Actually, one can easily see that this graph exactly corresponds to a term in Eq. (B.2), namely, to $2 \cdot 3 \cdot 1$, while there is a one-to-one correspondence between the pattern of sticks and every term there.

On the one hand side, when choosing $x + (x + 1)$ squares and putting (from the left side) a dot, a stick, a dot, etc. alternately on those squares, all possible ways of putting sticks and dots appear without repetition. Hence, the total number of all patterns is given by the number $\binom{n+x}{2x+1}$. On the other hand side, it is given by a sum of $\prod k_i$, as was mentioned above. Therefore, we get

$$\sum_{\{\sum k_i=n\}} k_1 k_2 \cdots k_{x+1} = \binom{n+x}{2x+1} . \quad (B3)$$

It is exactly the relation (4.8) we wanted, after replacing $n = p - j - 1$ and $x = j + 1$.

References

- [1] K. A. Intriligator and N. Seiberg, Nucl. Phys. Proc. Suppl. **45** (1996) 1 [hep-th/9509066]
- [2] L. Brink and J. H. Schwarz, Phys. Lett. **100B** (1981) 310
- [3] S. Ferrara and M. Lledo, JHEP **0005** (2000) 008 [hep-th/0002084];
D. Klemm, S. Penati and L. Tamasia, Class. and Quantum Grav. **20** (2003) 2905 [hep-th/0104190]
- [4] H. Ooguri and C. Vafa, Adv. Theor. Math. Phys. **7** (2003) 53 [hep-th/0302109];
ibid. **7** (2004) 405 [hep-th/0303063]
- [5] N. Seiberg, JHEP **0306** (2003) 010 [hep-th/0305248]
- [6] T. Hatanaka, S. V. Ketov, Y. Kobayashi and S. Sasaki, in preparation
- [7] G. Veneziano and S. Yankielowicz, Phys. Lett. **113B** (1982) 231
- [8] J. Wess and J. Bagger, *Supersymmetry and Supergravity*, Princeton University Press, 1992
- [9] S. J. Gates, Jr., S. V. Ketov and H. Nishino, Phys. Lett. **297B** (1992) 99 [hep-th/9203078]; *ibid.* **303B** (1993) 323 [hep-th/9203081]; Nucl. Phys. **B393** (1993) 149 [hep-th/9207042]
- [10] S. Ferrara and B. Zumino, Nucl. Phys. **B87** (1975) 207
- [11] S. J. Gates Jr., M. T. Grisaru, M. Roček and W. Siegel, *Superspace*, Front. Phys. **58** (1983) 1 [hep-th/0108200]

- [12] P. Fayet and S. Ferrara, Phys. Rep. **32** (1977) 251
- [13] F. Cachazo, M. R. Douglas, N. Seiberg and E. Witten, JHEP **0212** (2002) 071 [hep-th/0211170]
- [14] S. Arnone, F. Guerrieri and K. Yoshida, JHEP **0405** (2004) 031 [hep-th/0402035]
- [15] H. Kawai, T. Kuroki, T. Morita and K. Yoshida, *Direct derivation of the Veneziano-Yankielowicz superpotential from a matrix model*, hep-th/0412216
- [16] R. Dijkgraaf and C. Vafa, Nucl. Phys. **B644** (2002) 3 [hep-th/0206255]; Nucl. Phys. **B644** (2002) 21 [hep-th/0207106]; *A Perturbative Window into Non-Perturbative Physics*, hep-th/0208048
- [17] E. Ivanov, O. Lechtenfeld and B. Zupnik, JHEP **0402** (2004) 012 [hep-th/0308012]
- [18] S. V. Ketov and S. Sasaki, Phys. Lett. **595B** (2004) 530 [hep-th/0404119]; Phys. Lett. **597B** (2004) 105 [hep-th/0405278]; *$SU(2) \times U(1)$ non-anticommutative $N=2$ supersymmetric gauge theory*, hep-th/0407211, to appear in Int. J. Mod. Phys. **A** (2005)
- [19] J. de Boer, P. A. Grassi and P. van Nieuwenhuizen, Phys.Lett. **574B** (2003) 98 [hep-th/0302078]
- [20] M. R. Douglas and N. A. Nekrasov, Rev. Mod. Phys. **73** (2001) 977 [hep-th/0106048]
- [21] N. Seiberg and E. Witten, JHEP **9909** (1999) 032 [hep-th/9908142]
- [22] R. Britto, B. Feng and S.-J. Rey, JHEP **0307** (2003) 067 [hep-th/0306215]; S. Terashima and J.-T. Yee, JHEP **0312** (2003) 053 [hep-th/0306237]
- [23] A. Imaapur, JHEP **0309** (2003) 077 [hep-th/0308171];
P. A. Grassi, R. Ricci and D. Robles-Rana, JHEP **0407** (2004) 065 [hep-th/0311155];
R. Britto, B. Feng, O. Lunin and S.-J. Rey, Phys. Rev. **D69** (2004) 126004 [hep-th/0311275]
- [24] O. Lunin and S.-J. Rey, JHEP **0309** (2003) 045 [hep-th/0307275]

- [25] I. Jack, D.R.T. Jones and L.A. Worthy, *One-loop renormalisation of $N=1/2$ supersymmetric gauge theory*, hep-th/0412009;
S. Penati and A. Romagnoni, *Covariant quantization of $N=1/2$ SYM theories and supergauge invariance*, hep-th/0412041
- [26] M. Alishahiha, A. Ghodsi and N. Sadooghi, Nucl. Phys. **B691** (2004) 111 [hep-th/0309037]
- [27] O.D. Azorkina, A.T. Banin, I.L. Buchbinder and N.G. Plentev, *Generic chiral superfield model on nonanticommutative $N = \frac{1}{2}$ superspace*, hep-th/0502008.